



Solving Predator-Prey Model Using Maple 18 Coded Variational Iteration Method (VIM)

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Abstract

In this paper, MAPLE 18 codes are used to utilize Variational Iteration method for the numerical solution of two species Lotka-Volterra prey-predator interaction species which are governed by a system of nonlinear differential equations. Two examples are provided to show the ability and reliability of the method. The obtained approximate solution shows that Variational Iteration Method (VIM) is powerful numerical technique for solving a system of nonlinear differential equation, which can be easily applied to other nonlinear problems in biomathematics. This technique has shown to be very effective and yields accurate results.

1. Introduction

System of differential equation has a wide field in pure and applied mathematics such as transportation problem, economics mathematics, meteorology, biomathematics and engineering. All of these disciplines are concerned with the properties of differential equations of various types which emphasizes the rigorous justification and interpretation of natural phenomena. It plays an important role in modelling virtually every physical, technical, or biological process, from celestial motion, to bridge design, to interactions between neurons. Nonlinear differential equations are used for describing many phenomena in the real world as prey predator interactions. Prey predator models are classified as one of the most important applications in applied mathematics, thus many numerical and semi-analytical methods are developed for finding the solution of these problems by many researchers [1, 2, 3].

Relevant biological models may involve interactions (from the biochemical to the ecosystem level). One of the first interactions model in population dynamics was introduced in the beginning of the 20th century by Alfred Lotka, an American biophysicist (1925) and Vito Volterra, an Italian mathematician (1926) models interactions between preys and predators. The population sizes are denoted by $x(t)$ preys and $y(t)$ predators at time $t \geq 0$. He assumed that population change of one

specie depends on its current population, reproduction rate and interactions with other species (predator and prey) [4].

In recent years, research on Lotka-Volterra system of nonlinear differential equations have been extensively investigated in the literature. [5] proposed a new method for the explicit integration of Lotka-Volterra equations, the authors in [6] solved stochastic Lotka-Volterra equations via operational matrices, a study on competitive Lotka-Volterra model in random environments was examined and studied by [7]. [8] studied and observed the population dynamics of a three-species Lotka-Volterra model of two predators and their prey, [9] applied differential transformation method for solving prey predator model with holling type I. [10] studied a series solutions of two species Lotka-Volterra Equations by Adomian Decomposition and Homotopy Perturbation Methods and [11] studied Predator-Prey system in a polluted environment.

1.1 Lotka–Volterra Model

Lotka-Volterra model also known as the predator-prey equations, in deterministic subclasses, are well-known and have been an active area of research concerning ecological population modeling and economic modeling. These type of equations is so attractive in the terms of population dynamics of species competing (conflict) and the logistic population model. Thus, the Lotka–Volterra model in case of two species is a prey predator equation which is defined as follows:

$$\begin{cases} \frac{dx}{dt} = \alpha x(t) - \beta x(t)y(t) \\ \frac{dy}{dt} = -\omega y(t) + \tau x(t)y(t) \end{cases} \quad (1)$$

subject to initial conditions

$$\begin{cases} x(t_0) = A \\ y(t_0) = B \end{cases} \quad (2)$$

where the function $x(t)$ represents the populations of prey at time t , and also the function $y(t)$ represents the populations of predator at time t . All of the parameters $\alpha, \beta, \omega, \tau$ are non-negative constants. The parameter α represents the per capita reduction in prey per predator. The parameter β represents death rate per encounter of prey due to predation. Moreover if β is the only decreasing factor for the prey population, then prey will be eaten by predators. The parameter ω represents the per capita increase in predator per prey, moreover if ω is the only increasing factor for the predator population, then the population growth is proportional to the food available. The parameter τ represents mortality rate of predator and A, B are constants.

In reality, if the prey population is large, the predators will have more food to support a larger population. However, when the predator population grows too large, the prey begins to die off. This will result in a decrease in the predators. This trend continues as time goes on, implying a stable coexistence of the two populations.

The main objectives of this paper is to present and employ a MAPLE 18 software codes for variational iteration method proposed in [12] and to overcome the mathematical stress of integral involve in implementation of VIM.

This paper is organized as follows. In section 1, we introduced briefly the Lotka-Volterra model and their parameters notation related to Predator-prey interaction behaviors. Moreover, stability and equilibrium of the model are discussed. In section 2, we discussed and formulate MAPLE 18 Variation iteration scheme. Section 3, the variational iterative method was applied to solve predator-prey Equation (1) while in section 4, the numerical results table and graphs are reported. Finally, section 5 provides the discussion and conclusion.

1.2 The Dynamic Behavior of the Lotka-Volterra Model

One of the main properties of dynamic systems is stability. The stability is studied to determine some properties of solutions or system of differential equations. Consequently the dynamic behavior of the model will be discussed. For the model equilibrium points, we set the right hand side of (1) to zero.

$$\begin{aligned} (\alpha - \beta y(t))x(t) &= 0 \\ (-\omega + \tau x(t))y(t) &= 0 \end{aligned} \quad (3)$$

to obtain $x = 0, y = 0$ or $x = \frac{\omega}{\tau}, y = \frac{\alpha}{\beta}$. Then system (1) has

- i. the trivial equilibrium, $E_0(x_0, y_0) = (0, 0)$,
- ii. the non-trivial equilibrium point, $E^*(x^*, y^*) = \left(\frac{\omega}{\tau}, \frac{\alpha}{\beta}\right)$.

To investigate the stability of each equilibrium point, we evaluate the Jacobian matrix of (1) at each of these equilibrium. The Jacobian matrix of system (1) is given by

$$J(x, y) = \begin{pmatrix} \alpha - \beta y & -\beta x \\ \tau y & -\omega + \tau x \end{pmatrix}. \quad (4)$$

1.3 At the Trivial Equilibrium

We obtain from (4) that

$$J(E_0) = \begin{pmatrix} \alpha & 0 \\ 0 & -\omega \end{pmatrix}. \quad (5)$$

Thus the two eigenvalues of $J(E_0)$ are $\lambda_1 = \alpha$ and $\lambda_2 = -\omega$. Therefore, E_0 is a saddle point since one of the eigenvalues is positive and the other is negative.

The corresponding eigenvectors are the two axes x and y . Then using the standard computation of eigenvectors, we have for the eigenvector associated with $\lambda_1 = \alpha$

$$\begin{pmatrix} \alpha & 0 \\ 0 & -\omega \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \alpha \begin{pmatrix} x \\ y \end{pmatrix} \quad (6)$$

so that $(\alpha + \omega)y = 0$ and so, $y = 0$ which is the x –axis. Therefore, the x component of a perturbation away from but still near the E_0 will grow exponentially at a rate α .

Similarly, $x = 0$, which is the y – axis, is the eigenvector associated with $\lambda_2 = -\omega$. Therefore, the y component of a perturbation away from but still near the E_0 will shrink exponentially at a rate $-\omega$.

1.4 At the non-trivial equilibrium (Coexistence Equilibrium)

The Jacobian matrix in (4) becomes:

$$J(E^*) = \begin{pmatrix} 0 & -\beta \frac{\omega}{\tau} \\ \tau \frac{\alpha}{\beta} & 0 \end{pmatrix} \quad (7)$$

The eigenvalues of $J(E^*)$ are $\lambda_{1,2} = \pm i\sqrt{\alpha\omega}$. This show that the trace of this matrix is zero and the determinant is $\alpha\omega > 0$. Thus E^* is a centre and is balanced at the knife-edge between stable and unstable oscillations. Therefore system (1) is structurally unstable since any slight change to the structure of the equations, especially changing the nonlinear terms, could tip the balance between stability or instability, depending on how changes to the structure of the equations affects the real part of the eigenvalues. In a structurally unstable system, slight modifications to the form of the equations alter the stability.

The counterclockwise circulation of vectors near the coexistence equilibrium E^* is as a result of the purely imaginary eigenvalues of (E^*) . Any perturbation near this equilibrium causes the system to oscillate around it infinitely in a closed orbit, neither growing away from the equilibrium nor returning to it. The larger the perturbation, the greater the amplitude of the circulation, see Figures 4 and 6 for example.

2. Description of the Variation Iteration Method (VIM) and Solution Approach

The basic idea of the variational iteration method is to construct an iteration procedures based on correction functional that include a generalized Lagrange multiplier [13-14]. The VIM was proposed where the value of the multiplier was chosen using variational theory so that each improves the accuracy of the solution [15]. The initial approximation i.e. trial function usually includes unknown coefficient which can be determined to satisfy any boundary and initial conditions. VIM does not require specific transformation for nonlinear terms as required by other techniques and is now widely used by many researchers to study autonomous ordinary differential equation, Integro-differential systems, Linear Helmholtz partial differential equation and other fields [16-21]. In this method the solution is given in an infinite series usually convergent to an accurate solution. According to the variational iteration method we consider the following general differential equation of the form:

$$Lp + Np = g(t) \quad (8)$$

where L is a linear operator N is a nonlinear operator and $g(t)$ is an inhomogeneous term. We can construct a correctional function as follows

$$x_{n+1} = x_n(t) + \int_0^t \lambda \{Lx_n(s) + N\tilde{x}_n(s) - g(s)\} ds \quad (9)$$

Where λ is a Lagrangian multiplier which can be identified optimally via variational theory [22]. The subscript n denotes the n th approximation Consider the stationary condition of the above correction functional then the Lagrange multiplier can be expressed as

$$\lambda_i(w) = \frac{(-1)^q}{(q-1)!} (q-t)^{q-1} \quad (10)$$

Where q is the highest order of the differential equation.

2.1 Maple 18 Coded Variational Iteration Method

In order to formulate the general variational iteration approach on MAPLE 18 software, we consider Equations (1) and (2) and develop the VIM schemes as follows:

MAPLE 18 Coded Variational Iteration Scheme

$$\left. \begin{aligned}
 & \text{restart:} \\
 & \alpha := R_1; \beta := R_2; \omega := R_3; \tau := R_4; \lambda := (-1); \\
 & x_0 := A; y_0 := B; a_0 := \text{diff}(x_0, t); a_1 := \text{diff}(y_0, t); \\
 & x_1 := x_0 + (\lambda) * \text{int}\{(a_0 - (\alpha * x_0) + (\beta * x_0 * y_0)), t = 0 \dots t\}; \\
 & \quad x_{11} := \text{value}(x_1); b_0 := \text{diff}(x_{11}, t); \\
 & y_1 := y_0 + (\lambda) * \text{int}\{(a_1 + (\omega * y_0) - (\tau * x_0 * y_0)), t = 0 \dots t\}; \\
 & \quad y_{11} := \text{value}(y_1); b_1 := \text{diff}(y_{11}, t); \\
 & x_2 := x_{11} + (\lambda) * \text{int}\{(b_0 - (\alpha * x_{11}) + (\beta * x_{11} * y_{11})), t = 0 \dots t\}; \\
 & \quad x_{12} := \text{value}(x_2); c_0 := \text{diff}(x_{12}, t); \\
 & y_2 := y_{11} + (\lambda) * \text{int}\{(b_1 + (\omega * y_{11}) - (\tau * x_{11} * y_{11})), t = 0 \dots t\}; \\
 & \quad y_{12} := \text{value}(y_2); c_1 := \text{diff}(y_{12}, t); \\
 & x_3 := x_{12} + (\lambda) * \text{int}\{(c_0 - (\alpha * x_{12}) + (\beta * x_{12} * y_{12})), t = 0 \dots t\}; \\
 & \quad x_{13} := \text{value}(x_3); d_0 := \text{diff}(x_{13}, t); \\
 & y_3 := y_{12} + (\lambda) * \text{int}\{(c_1 + (\omega * y_{12}) - (\tau * x_{11} * y_{12})), t = 0 \dots t\}; \\
 & \quad y_{13} := \text{value}(y_3); d_1 := \text{diff}(y_{13}, t); \\
 & \quad \vdots \\
 & \quad \vdots \\
 & x_m := x_{1m-1} + (\lambda) * \text{int}\{(s_0 - (\alpha * x_{1m-1}) + (\beta * x_{1m-1} * y_{1m-1})), t = 0 \dots t\}; \\
 & \quad x_{1m} := \text{value}(x_m); z_0 := \text{diff}(x_{1m}, t); \\
 & y_m := y_{1m-1} + (\lambda) * \text{int}\{(s_1 + (\omega * y_{1m-1}) - (\tau * x_{1m-1} * y_{1m-1})), t = 0 \dots t\}; \\
 & \quad y_{1m} := \text{value}(y_m); z_1 := \text{diff}(y_{1m}, t); \\
 & \quad x(t) := \text{evalf}(x_{1m}); \\
 & \quad y(t) := \text{evalf}(y_{1m});
 \end{aligned} \right\} \quad (11)$$

where m is the number of iterations, R_1, R_2, R_3, R_4 are parameter constants and A, B are constants. Thus, $x(t)$ and $y(t)$ are the series solutions at m th iteration.

3. Numerical Experiment

In this section, we examine the predator and prey growth/decay of the two species (self-interaction) as well as their interaction. We investigate behaviours of the parameters α, β, ω and τ which play a key role in determining the system behaviours. Two examples are considered subject to specific initial condition of the model. Using the MAPLE 18 coded Variation Iteration scheme, we obtained the numerical solution for Equation (1) and initial condition (2) at iteration step $m = 10$ as shown in Tables 1 to 6.

Example 1 Suppose α is the per capita reduction in prey per predator and β is the death rate per encounter of prey due to predation [23].

$$\left\{ \begin{array}{l} \text{when } (\alpha < \beta) \\ \alpha = 3.029850 \\ \beta = 4.094132 \\ \omega = 1.967217 \\ \tau = 2.295942 \\ \lambda = -1.00000 \\ x_0 = 1.187100 \\ y_0 = 0.740047 \end{array} \right. \left\{ \begin{array}{l} \text{when } (\alpha > \beta) \\ \alpha = 4.094132 \\ \beta = 3.029850 \\ \omega = 1.967217 \\ \tau = 2.295942 \\ \lambda = -1.00000 \\ x_0 = 1.187100 \\ y_0 = 0.740047 \end{array} \right. \left\{ \begin{array}{l} \text{when } (\alpha = \beta) \\ \alpha = 3.029850 \\ \beta = 3.029850 \\ \omega = 1.967217 \\ \tau = 2.295942 \\ \lambda = -1.00000 \\ x_0 = 1.187100 \\ y_0 = 0.740047 \end{array} \right.$$

Substitute the above parameters into algorithm (11), we have the following solutions:

Table 1: ($\alpha < \beta$)

t	Analytical Solution $x(t)$	VIM Solution $x(t)$	Analytical Solution $y(t)$	VIM Solution $y(t)$
0.0	1.187100000000	1.187100000000	0.740047000000	0.740047000000
0.1	1.173293314174	1.173293331000	0.7975048380086	0.7975047610000
0.2	1.132681174388	1.132681233000	0.8540331977419	0.8540330912000
0.3	1.069749554336	1.069749539000	0.9036493434085	0.9036492527000
0.4	0.992376686741	0.992376610300	0.9407098212139	0.9407097698000
0.5	0.909715706923	0.909715551500	0.9612974253317	0.9612976897000
0.6	0.829963573965	0.829961925000	0.9640576734517	0.9640598752000
0.7	0.758989599935	0.758979098800	0.9501583685533	0.9501676352000
0.8	0.700090581059	0.700045711900	0.9225636508131	0.9225907487000
0.9	0.654510497041	0.654363278700	0.8850829288848	0.8851423567000
1.0	0.622205729269	0.621805168400	0.8415741110666	0.8416743875000

Table 2: ($\alpha > \beta$)

t	Analytical Solution $x(t)$	VIM Solution $x(t)$	Analytical Solution $y(t)$	VIM Solution $y(t)$
0.0	1.187100000000	1.187100000000	0.740047000000	0.740047000000
0.1	1.413308431481	1.413308491000	0.8192192423347	0.819219212200
0.2	1.629422801606	1.629422522000	0.9548299457312	0.954830107800
0.3	1.783965142331	1.783964837000	1.1626432476204	1.162643502000
0.4	1.812364112012	1.812363259000	1.4474459753223	1.447445942000
0.5	1.674587231402	1.674575344000	2.0850176428230	2.085017642720
0.6	1.401596690416	1.401562584000	2.0850176428292	2.085017641320
0.7	1.085902388192	1.086180815000	2.2783956366840	2.278395618000
0.8	0.811351279883	0.811351460690	2.3239540305871	2.323951416000
0.9	0.610020458917	0.610020361200	2.2440744293156	2.244074124000
1.0	0.476079350030	0.476079236100	2.0857367907384	2.085736623000

Table 3: ($\alpha = \beta$)

t	Analytical Solution $x(t)$	VIM Solution $x(t)$	Analytical Solution $y(t)$	VIM Solution $y(t)$
0.0	1.187100000000	1.187100000000	0.740047000000	0.740047000000
0.1	1.272187665251	1.272187275000	0.806474171253	0.806474407600
0.2	1.331974163143	1.331973873000	0.893823428997	0.893823624800
0.3	1.354102768524	1.354102583000	1.000208677394	1.000208771000
0.4	1.330095316143	1.330095126000	1.119123022586	1.119122920000
0.5	1.259716623148	1.259716980000	1.238564988790	1.238564225000
0.6	1.152840256063	1.152845461000	1.342772096989	1.342768205000
0.7	1.026594271894	1.026627603000	1.416793455406	1.416793581000
0.8	0.899120517439	0.899244376100	1.451520906161	1.451521131000
0.9	0.784059292436	0.784362541500	1.445978046355	1.445913412000
1.0	0.688601146327	0.688090398100	1.405950208844	1.405941241000

Example 2 Suppose α is the per capita reduction in prey per predator and β is the death rate per encounter of prey due to predation [24].

$$\left\{ \begin{array}{l} \text{when } (\alpha > \beta) \\ \alpha = 0.100 \\ \beta = 0.0014 \\ \omega = 0.080 \\ \tau = 0.0012 \\ \lambda = -1.000 \\ x_0 = 4.000 \\ y_0 = 9.000 \end{array} \right. \quad \left\{ \begin{array}{l} \text{when } (\alpha < \beta) \\ \alpha = 0.0014 \\ \beta = 0.100 \\ \omega = 0.080 \\ \tau = 0.0012 \\ \lambda = -1.000 \\ x_0 = 4.000 \\ y_0 = 9.000 \end{array} \right. \quad \left\{ \begin{array}{l} \text{when } (\alpha = \beta) \\ \alpha = 0.100 \\ \beta = 0.100 \\ \omega = 0.080 \\ \tau = 0.0012 \\ \lambda = -1.000 \\ x_0 = 4.000 \\ y_0 = 9.000 \end{array} \right.$$

Substitute the above parameters into algorithm (11), we have the following solutions:

Table 4: ($\alpha > \beta$)

t	Analytical Solution $x(t)$	VIM Solution $x(t)$	Analytical Solution $y(t)$	VIM Solution $y(t)$
0.0	4.000000000000	4.000000000000	9.000000000000	9.000000000000
0.1	4.035132287030	4.035132286000	8.932592638256	8.932592638000
0.2	4.070611403006	4.070611402000	8.865727698739	8.865727700000
0.3	4.106440756278	4.106440756000	8.799400925152	8.799400926000
0.4	4.142623789899	4.142623790000	8.733608094714	8.733608095000
0.5	4.179163981615	4.179163982000	8.668345018151	8.668345018000
0.6	4.216064844061	4.216064844000	8.603607539534	8.603607540000
0.7	4.253329953021	4.253329924000	8.539391509681	8.539391537000
0.8	4.290962896869	4.290962804000	8.475692832270	8.475692921000
0.9	4.328967276114	4.328967104000	8.412507466043	8.412507636000
1.0	4.367346731478	4.367346480000	8.349831399216	8.349831655000

Table 5: ($\alpha < \beta$)

t	Analytical Solution $x(t)$	VIM Solution $x(t)$	Analytical Solution $y(t)$	VIM Solution $y(t)$
0.0	4.000000000000	4.000000000000	9.000000000000	9.000000000000
0.1	3.657473020744	3.657473258000	8.932387296668	8.932387294000
0.2	3.346536796070	3.346536998000	8.864935239948	8.864935237000
0.3	3.064097573818	3.064097837000	8.797679574554	8.797679571000
0.4	2.807379952412	2.807380177000	8.730651950326	8.730651947000
0.5	2.573892073952	2.573892359000	8.663880372174	8.663880368000
0.6	2.361396256269	2.361396506000	8.597389579842	8.597389576000
0.7	2.167881361179	2.167881643000	8.531201405234	8.531201402000
0.8	1.991539481074	1.991539751000	8.465335073997	8.465335070000
0.9	1.830744135389	1.830744404000	8.399807487591	8.399807484000
1.0	1.684031442829	1.684031744000	8.334633466917	8.334633463000

Table 6: ($\alpha = \beta$)

t	Analytical Solution $x(t)$	VIM Solution $x(t)$	Analytical Solution $y(t)$	VIM Solution $y(t)$
0.0	4.000000000000	4.000000000000	9.000000000000	9.000000000000
0.1	3.693713901517	3.693714070000	8.932407277368	8.932407275000

0.2	3.413183652004	3.413183941000	8.865010279466	8.865010275000
0.3	3.156082329242	3.156082424000	8.797838192760	8.797838190000
0.4	2.920304355828	2.920304706000	8.730917063216	8.730917058000
0.5	2.703946085963	2.703946207000	8.664270073277	8.664270071000
0.6	2.505282320509	2.505282688000	8.597917875002	8.597917871000
0.7	2.322752234866	2.322752401000	8.531878791541	8.531878789000
0.8	2.154939736038	2.154940050000	8.466169095953	8.466169091000
0.9	2.000562151892	2.000562388000	8.400803173218	8.400803170000
1.0	1.858455074810	1.858455294000	8.335793735914	8.335793731000

4. Graphs Representation

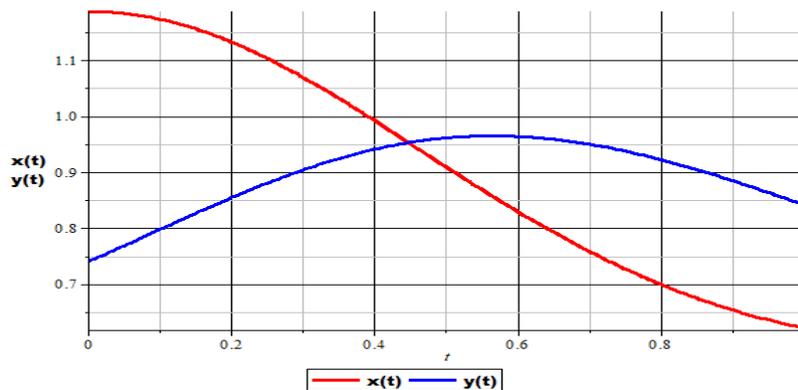


Figure 1 Numerical solutions of the Predator $y(t)$ and Prey $x(t)$ when $\alpha < \beta$ Example 1

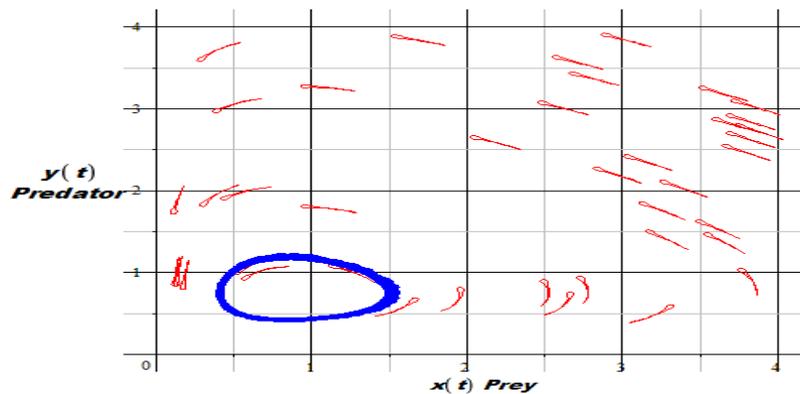


Figure 2 Unsteady states of phase planes for Predator-Prey when $\alpha < \beta$ Example 1

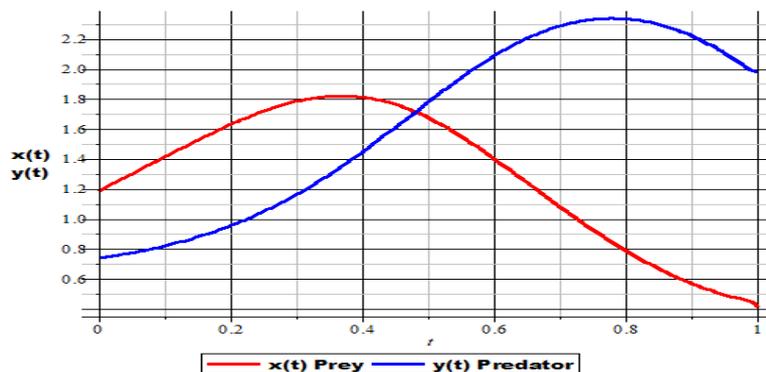


Figure 3 Numerical solutions of the Predator $y(t)$ and Prey $x(t)$ when $\alpha > \beta$ Example 1

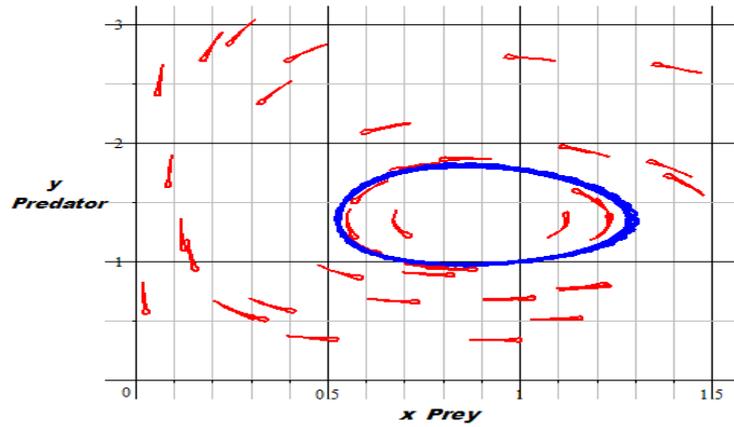


Figure 4 Steady states of phase planes for Predator-Prey when $\alpha > \beta$ Example 1

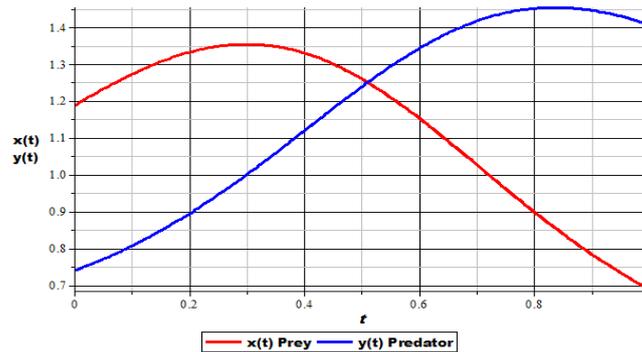


Figure 5 Numerical solutions of the Predator $y(t)$ and Prey $x(t)$ when $\alpha = \beta$ Example 1

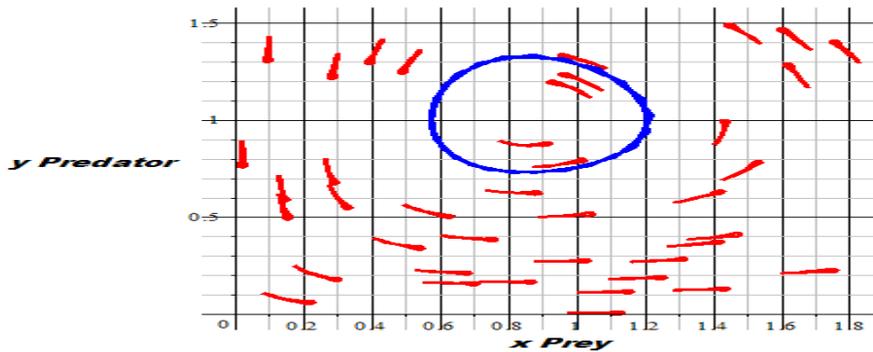


Figure 6 Steady states of phase planes for Predator-Prey when $\alpha = \beta$ Example 1

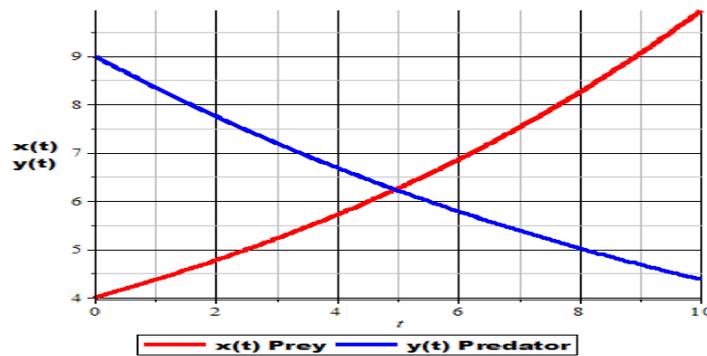


Figure 7 Numerical solutions of the Predator $y(t)$ and Prey $x(t)$ when $\alpha > \beta$ Example 2

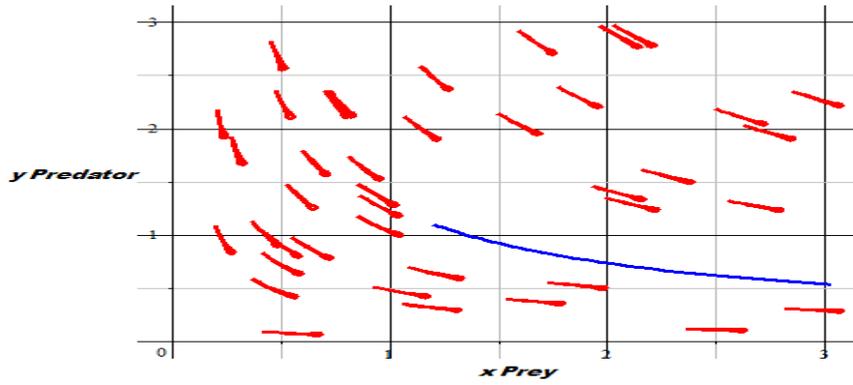


Figure 8 Unmutualistic interactions phase for Predator-Prey when $\alpha > \beta$ Example 2

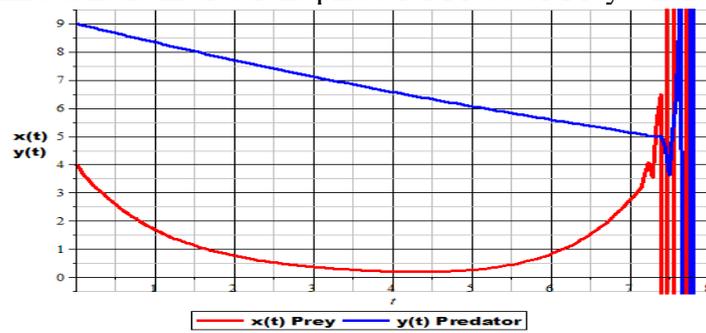


Figure 9 Numerical solutions of the Predator $y(t)$ and Prey $x(t)$ when $\alpha < \beta$ Example 2

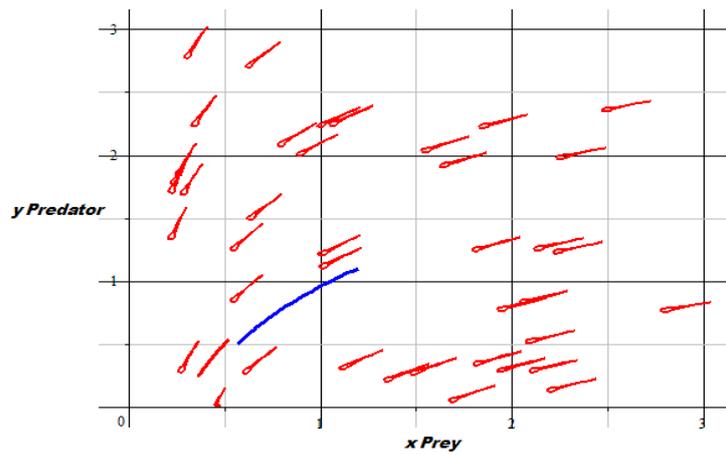


Figure 10 Mutualistic interactions phase for Predator-Prey when $\alpha < \beta$ Example 2

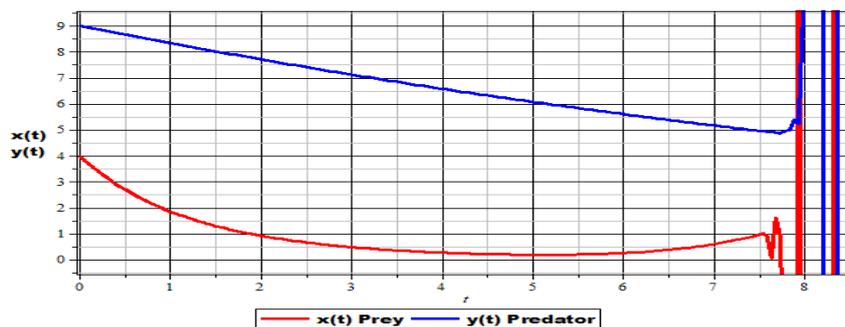


Figure 11 Numerical solutions of the Predator $y(t)$ and Prey $x(t)$ when $\alpha = \beta$ Example 2

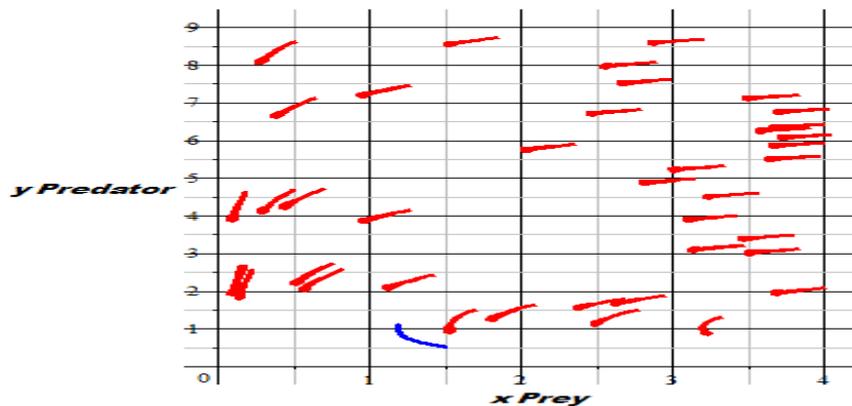


Figure 12 Unmutualistic interactions phase for Predator-Prey when $\alpha = \beta$ Example 2

3.1 Discussion

The numerical solutions for six cases of interaction behaviors between Predator-Prey are presented in Tables 1 to 6 while Figures 1 to 6 show plots relationship of Predator-Prey for example one and Figures 7 to 12 depict the numerical solution and interaction behaviors of Predator-Prey for example two. Finally, the study have revealed the interactions behaviors between two species (steady, unsteady, mutualistic and unmutualistic interactions) from computational analysis.

4. Conclusion

This article highlights the feasibility and capability of MAPLE 18 software codes to solve nonlinear system of Lotka–Volterra two species of Predator-Prey model. The conventional variation iteration method was employed using MAPLE 18 software commands to overcome the rigorous computational work and simplification of integrals involves during iteration process. Two examples are consider to test the efficiency of the algorithm and the numerical results obtained were compared with analytical solutions with little relative errors. The advantage of the MCVIM over the conventional approach is faster, easy and it provides an efficient numerical solutions. Finally, the proposed scheme is easy to implement and shows a good agreement with analytic results.

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