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# Optimisation of Cut-Off Grade Considering Dilution and Waste Rehabilitation Costs

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**Abstract**

*Cut-off grade is one of the most important factors in open pit mine production planning. Due to the sensitivity of this parameter, which can greatly affect the net present value (NPV) and cash flow of mineral projects, its accuracy is of great importance. To determine the optimal cut-off grade, Lane's method is commonly used. Lane provided his method to determine the optimal cut-off grade by considering factors such as the capacity of the mine, treatment plant and refinery facilities, the time value of money and grade distribution of the mineral deposit. In Lane's algorithm, mining dilution and waste rehabilitation as well as their associated costs are not considered during the optimisation. In this work, the effects of dilution and waste rehabilitation on the cut-off grade are studied using Lane's theory. These costs are inserted directly into Lane's objective function and production schedules are drawn. Results showed that dilution and waste rehabilitation, when considered in cut-off grade policies, decreased the average grade of materials and consequently, cut-off grade increased. As a result, the yearly processed tonnage and amount refined decreased in a significant portion of the mine life. This led to a reduction in the annual profits and NPVs by $ 2 388 (a 28% decrease) and $ 944 (a 37% decrease) respectively.*

## 1. Introduction

Cut-off grade can be defined as the level of ore grade used to separate two courses of action: mine or to leave the unmined, mill, or dump [1]. A material whose content is above the specified cut-off grade is treated as ore, and that whose content is below the cut-off grade is treated as waste. A concept from [2] defines cut-off grade as a grade that defines the destination of a given quantity of material – brought to the surface or left in the ground. Materials brought to the surface can be sent to the mill, the waste dump, or the lean ore dump. Materials can also be stored on leach pads for further treatment. All of these destinations are chosen depending on the mining method and the prevailing economic conditions.

Cut-off grade is crucial in mining and is used as a tool for deciding the fate of a block in economic terms during mine planning [3]. It defines what is milled, left in place, sent to the waste dump and stockpiled. The choice of cut-off grades influences the profitability of mining projects and the quantities determine the amount of material that can be extracted economically [4]. In other words, cut-off grade values directly impact the profits of mining operations and, consequently, the net present value (NPV).

The cut-off grade is adjusted as the economic environment changes in terms of metal prices and mining costs, and it is always changing. A high cut-off grade can be used to boost a project's short-term profitability and net present value (NPV), potentially increasing the benefit to shareholders and other financial stakeholders, such as the government and local communities [5].

However, increasing the cut-off grade is likely to shorten the mine's life. This could have several consequences on the mine and other stakeholders at large. A shorter mine life can reduce time-dependent opportunities, such as price cycles. A shorter mine life can also have a more significant socioeconomic impact due to decreased long-term employment and benefits to employees and local communities.

Increased cut-off grades may reduce political risk by ensuring a higher financial return in a shorter period. When commodity prices rise, the cut-off grade may be raised to strengthen the company's financial position and reduce the risk of failure when commodity prices go down [5].

On the other hand, during periods of high prices, cut-off grades may be reduced to increase mine life and allow for storage of high-grade materials to maintain a good profit level when commodity prices fall. Economic or technical performance criteria imposed by bank loans and other financial institutions may also constrain cut-off grades [5].

In all cases, considering the dependence on economic parameters such as metal price, mining and milling costs, and other fundamental issues, the selection of cut-off grades to maximise NPV over the life of mine becomes very critical [6]. The objective most widely accepted in cut-off grade optimisation studies is to maximise the NPV of future cash flows.

Cut-off grade theory dates as far as the early 1950s.Viewpoint in [7] bases economic cut-off grade as a function of only costs. He defined the cut-off grade as the lowest grade at which a parcel of material mined can pay for its costs.

However, [8] coined formulas that determine cut-off grades based on a break-even approach, and he introduced declining cut-off grade policies. This was followed by Lane's contribution, which captured the capacities of the various components of the mining system with the grade distribution and the different costs associated with mining.

Several works after Lane focused on modifying Lane's approach to suit current situations in the mining industry. A proposal in [1] came up with an approach similar to Lane's, with a slight difference in calculating the balancing cut-off grades.

Results from [[9] included the option of stockpile in applying cut-off grades to multimineral deposits. Proposals in [10] employed equivalent grade factors to determine the cut-off grades for multiple metal deposits in open pit mines. Suggestions from [11;12] modified Lane's algorithm by considering commodity price, cost escalation, and stockpiling options for mining two economic minerals.

The outcome of [13] also introduced an "optimisation factor" based on the generalised reduced gradient algorithm to maximise the NPV of a project. [14] optimised cut-off grades by modifying Lane's cut-off grade theory with mixed-integer linear programming. Rehabilitation cost analysis in [15] determined that the costs of mining and dumping waste constitute a portion of the rehabilitation costs and must thus be included in the cut-off grade calculation.

Variable capacities determination in open pit in[16] proposed an algorithm investigating the concept of variable capacities on the three major limiting factors: mine, mill, and market. Their method attempts to improve Lane's original algorithm by calculating mining, processing, and market capacities as variable parameters. A heuristic approach to stochastic cut-off grade optimisation in [17] successfully applied Lane's theory to a mine with multiple processing streams. An outcome in [18] also showed that the Lane's algorithm could yield much more realistic cut-off grades when dilution is considered.

Given the significant negative impacts of dilution and other environmental policies like waste rehabilitation on the entire mining process and its parameters, it is important to consider these components and their costs during the cut-off grade optimization process. It is the objective of this paper to determine the optimum cut-off grade with consideration to dilution and waste rehabilitation cost.

## 2. Materials and Method

### 2.1 Lane's Algorithm

In 1964, Kenneth F. Lane presented a seminar paper entitled "choosing the optimal cut-off grade", in which he developed an algorithm for cut-off optimisation [19]. It has been applied in various software packages like Maptek Evolution® and Whittle 4X®. This algorithm has been used as the basis for most research works in optimising cut-off grades. Cut-off grade optimisation based on Lane's algorithm divides a mining system into three (3) stages, namely: mining stage, milling stage (processing stage) and refinery stage.

Each stage is associated with a cost and a limiting capacity. Lane finds six (6) cut-off grades as candidates for finding the optimised cut-off grade. These six (6) cut-off grades comprise three (3) limiting cut-off grades by economics and the other three (3) cut-off grades by balancing operations.

Lane uses sorting and middle-value criteria to select the optimum cut-off grade among these six (6) cut-off grades. Given the optimum, the yearly profits and net present value (NPV) can be obtained. It is worth mentioning that the chosen cut-off grade affects profits and net present value (NPV) of operations hence maximisation of the NPV, Lane's objective. The profit equation according to Lane is as follows [19:

$$P = (s - r)Q_r - cQ_c - mQ_m - fT \quad (1)$$

The cost of dilution and waste rehabilitation would be incorporated into Lane's algorithm. In this case, unlike Lane's method, $Q_c$ is not solely determined by the grade; rather, the grade and dilution work together to determine $Q_c$ and $Q_r$. Also, the rehabilitation cost applies to all materials that are dumped as waste ($Q_m$ - $Q_c$). With the parameters already defined, the basic equations are given below. The new total cost Tc is defined in Equation (2) as:

$$Tc = mQ_m + cQ_c + rQ_r + fT + h(Q_m - Q_c) + c_DQ_c + r_DQ_r \quad (2)$$

where,
$h$ = the rehabilitation cost, which is the cost per ton of rehabilitating material of a particular rock type after it has been dumped as waste.
$c_D$ = additional costs incurred as a result of dilution in terms of dollars per ton milled.
$r_D$ = dilution costs in terms of dollars per unit of product incurred at the product and selling stages.

Rearranging the terms in Equation (2) results in Equation (3) as:

$$Tc = (m + h)Q_m + (c + c_D)Q_c + (r + r_D)Q_r + fT \tag{3}$$

The revenue R, is given as

$$R = sQ_r \tag{4}$$

Profit given as $P = \text{Revenue}(R) - \text{Total Cost}(Tc)$ is expressed in Equation (5) as:

$$P = sQ_r - ((m + h)Q_m + (c + c_D)Q_c + (r + r_D)Q_r + fT)) \tag{5}$$

Grouping the terms results in Equation (6).

$$P = (s - r - r_D)Q_r - (c + c_D)Q_c - (m + h)Q_m - fT \tag{6}$$

**Net Present Value Maximisation**

In today's mining operations, the objective of cut-off grade optimisation is to maximise the NPV. The objective function of the optimisation process with regard to the NPV maximisation proposed by Lane is given as follows:

$$v = (s-r)Q_r - cQ_c - mQ_m - fT - VdT \tag{7}$$

Accounting for dilution and waste rehabilitation costs results in Equation (8) as:

$$v = (s - r - r_D)Q_r - (c + c_D)Q_c - (m + h)Q_m - T(f + Vd) \tag{8}$$

Therefore, taking each operation as the limiting constraint in Equation (8), the resulting equations are given in Equations (9, 10 and 11) as:

$$v_m = (s - r - r_D)Q_r - (c + c_D)Q_c - \left(m + h + \frac{f + Vd}{M}\right)Q_m \tag{9}$$

$$v_c = (s - r - r_D)Q_r - (m + h)Q_m - \left(c + c_D + \frac{f + Vd}{C}\right)Q_c \tag{10}$$

$$v_r = \left(s - r - r_D - \frac{f + Vd}{R}\right)Q_r - (c + c_D)Q_c - (m + h)Q_m \tag{11}$$

The curve $v_m$ represents the increase in NPV due to the mine being a bottleneck in the system, and the grade at which $v_m$ is maximum corresponds to the mine limiting cut-off grade ($g_m$). The curve $v_c$ represents the increase in NPV due to the concentrator being a bottleneck in the system, and the grade at which $v_c$ is maximum corresponds to the mill limiting cut-off grade ($g_c$). The curve $v_r$ represents the increase in NPV due to the refinery being a bottleneck in the system, and the grade at which $v_r$ is maximum corresponds to the refinery or market limiting cut-off grade ($g_r$). All plots have upward convexity [6]. Lane's cut-off policy generates a cut-off grade policy for the whole mine life. Every year, it shows optimum cut-off grade (G), quantities ($Q_m$, $Q_c$ and $Q_r$), profit; and NPV. This is done through an iterative process until the value V converges.

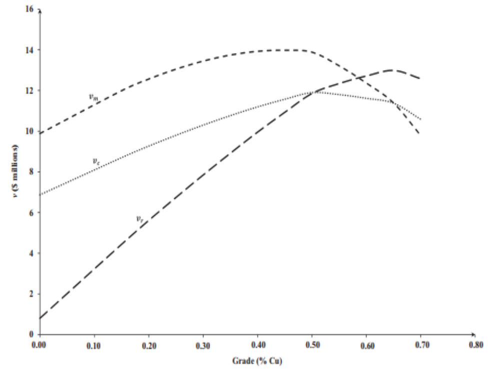


**Figure 1 $v_m$, $v_r$ and $v_c$ as a Function of g, Source: [20]**

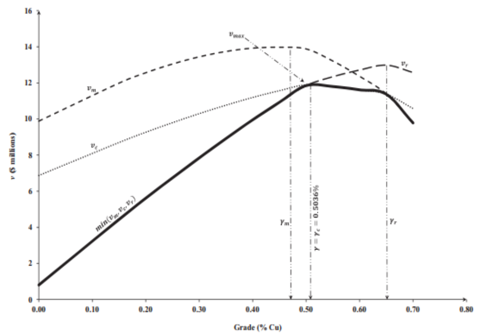


**Figure 2 Choosing the Optimum Cut-off in Lane's Model, Source: [20]**

## 4.1 Data Collection

To show the effect of dilution and waste rehabilitation costs on the optimum cut-off grade in this section, an example data from [2] is used. A final pit that contains 1000 tonnes of materials was superimposed on a mineral inventory, and the result is shown in Table 1.

**Table 1 Initial Mineral Inventory**

| **Grade Categories (grams/Tonne)** | | **Quantity (Q)** |
|---|---|---|
| **From** | **To** | **Tonnes** |
| 0 | 0.1 | 100 |
| 0.1 | 0.2 | 100 |
| 0.2 | 0.3 | 100 |
| 0.3 | 0.4 | 100 |
| 0.4 | 0.5 | 100 |
| 0.5 | 0.6 | 100 |
| 0.6 | 0.7 | 100 |
| 0.7 | 0.8 | 100 |
| 0.8 | 0.9 | 100 |
| 0.9 | 1.0 | 100 |
| $Q_m$ | | **1000** |

**(Source: [2])**

The economic parameters and operational capacities applicable at the time of evaluation are also given in Table 2.

**Table 2 Economic Parameters and Operational Capacities**

| **Parameter** | **Symbol** | **Value** |
|---|---|---|
| **Price** | | |
| Price of Commodity | s | $25/gram |
| **Capacities** | | |
| Mining Capacity | M | 100 tonnes/year |
| Processing Capacity | C | 50 tonnes/year |
| Refining Capacity | R | 40 tonnes/year |
| **Costs** | | |
| Mining cost | m | $ 1/tonne |
| Processing cost | c | $ 2/tonne |
| Refining Cost | r | $ 5/tonne |
| Fixed Cost | f | $ 300/year |
| **Percentages** | | |
| Recovery | y | 100% |
| Discount | d | 15% |

The basic objective of optimising cut-off grades is to maximise the NPV of the mining system, as discussed earlier. To maximise NPV, one must maximise the increase in present values ($v_m$, $v_c$, and $v_r$) as derived in Equations (9, 10 and 11).

For a dilution factor of 10%, 100 tons of waste material will be added to the volume of material that will be mined (assuming that the waste material's grade is zero). The new grade distribution as a result of dilution is shown in Table 3.

**Table 3 Initial Mineral Inventory Due to Dilution**

| **Grade Categories (grams/Tonne)** | | **Quantity (Q)** |
|---|---|---|
| **From** | **To** | **Tonnes** |
| 0 | 0.09 | 100 |
| 0.09 | 0.18 | 100 |
| 0.18 | 0.27 | 100 |
| 0.27 | 0.36 | 100 |
| 0.36 | 0.45 | 100 |
| 0.45 | 0.54 | 100 |
| 0.54 | 0.63 | 100 |
| 0.63 | 0.72 | 100 |
| 0.72 | 0.81 | 100 |
| 0.81 | 0.90 | 100 |
| 0.90 | 0.99 | 100 |
| Qm | | **1100** |

**Determining the Cut-off Grade Policy for Various Years**

Cut-off grades are optimised assuming that all costs incurred by dilution at the concentrating and refining stages ($c_D$ and $r_D$) are 10% of the initial concentration and refining costs. Waste rehabilitation cost is assumed to be $ 0.5/tonne.

An initial iteration is performed where the value of V is set to 0, and $v_m$, $v_c$, and $v_r$ are plotted as a function of the cut-off grade. The results are displayed in Table 4 and Figure 3

**Table 4 $v_m$, $v_c$, and $v_r$ as a Function of Cut-off Grades For Year 1 (V=0)**

| g | $g_{avg}$ | $Q_m$ | $Q_c$ | $Q_r$ | $v_m$ | $v_c$ | $v_r$ |
|---|---|---|---|---|---|---|---|
| 0 | 0.5 | 1000 | 1000 | 500 | 4000 | 1000 | 3250 |
| 0.1 | 0.55 | 1000 | 900 | 495 | 4100 | 1700 | 3387.5 |
| 0.2 | 0.6 | 1000 | 800 | 480 | 4000 | 2200 | 3400 |
| 0.3 | 0.65 | 1000 | 700 | 455 | 3700 | 2500 | 3287.5 |
| 0.4 | 0.7 | 1000 | 600 | 420 | 3200 | 2600 | 3050 |
| 0.5 | 0.75 | 1000 | 500 | 375 | 2500 | 2500 | 2687.5 |
| 0.6 | 0.8 | 1000 | 400 | 320 | 1600 | 2200 | 2200 |
| 0.7 | 0.85 | 1000 | 300 | 255 | 500 | 1700 | 1587.5 |
| 0.8 | 0.9 | 1000 | 200 | 180 | -800 | 1000 | 850 |
| 0.9 | 0.95 | 1000 | 100 | 95 | -2300 | 100 | -12.5 |

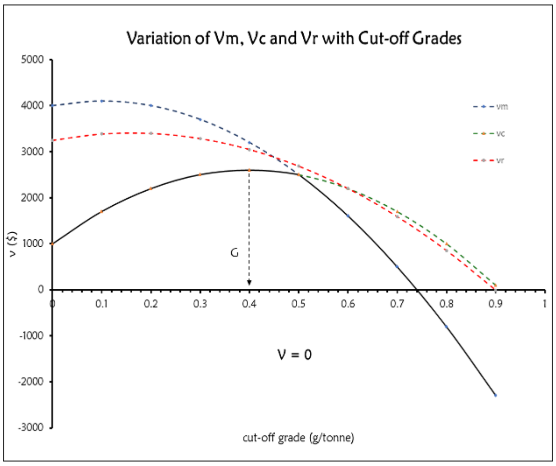


**Figure 3 Choosing Optimum Cut-off Grade for Year 1 (Initial Iteration, V=0)**

The optimum cut-off grade is the grade at which the optimum curve is maximum, thus at 0.425 g/t. Using Table 3, the tonnes of ore ($T_o$), tonnes of waste ($T_w$), stripping ratio (S.R) and average grade ($g_{avg}$) are calculated at the chosen cut-off, thus 0.425 g/t. The NPV obtained is $ 816.55. This new value of V ($ 816.55) is used to calculate the optimal cut-off grade with the corresponding quantities and the total NPV in the first iteration.

The value of V is set to $ 816.55, and $v_m$, $v_c$, and $v_r$ are again plotted as a function of cut-off grade. The process is repeated over and over again until the NPV converges, thus the calculated V becomes the same as the NPV in the previous iteration.

The next step is to adjust the grade tonnage distribution curve and find the optimum cut-off grades for the subsequent years. The process is repeated until the whole deposit is depleted. The production schedule obtained is given in Table 6 as against that from the original Lane's algorithm in Table 5.

**Table 5 Final Production Schedule from Lane's Algorithm**

| Year | Cut-off grade (g/t) | $Q_m$ (t) | $Q_c$ (t) | Avg Grade (g/t) | $Q_r$ (g) | Profit ($) | NPV ($) |
|---|---|---|---|---|---|---|---|
| 1 | 0.50 | 100 | 50 | 0.75 | 37.5 | 250 | 1,255 |
| 2 | 0.50 | 100 | 50 | 0.75 | 37.5 | 250 | 1 193 |
| 3 | 0.50 | 100 | 50 | 0.75 | 37.5 | 250 | 1 122 |
| 4 | 0.50 | 100 | 50 | 0.75 | 37.5 | 250 | 1 040 |
| 5 | 0.50 | 100 | 50 | 0.75 | 37.5 | 250 | 946 |
| 6 | 0.50 | 100 | 50 | 0.75 | 37.5 | 250 | 838 |
| 7 | 0.50 | 100 | 50 | 0.75 | 37.5 | 250 | 714 |
| 8 | 0.49 | 97 | 50 | 0.74 | 37.1 | 245.7 | 574 |
| 9 | 0.46 | 93 | 50 | 0.73 | 36.55 | 238 | 417 |
| 10 | 0.44 | 89 | 50 | 0.72 | 35.9 | 229 | 243 |
| 11 | 0.40 | 21 | 12.6 | 0.70 | 8.8 | 55 | 53 |

**Source: [2]**

**Table 6 Final Production Schedule from Lane's Algorithm with Dilution and Waste Rehabilitation Cost**

| Year | Cut-off grade g/t) | $Q_m$ (t) | $Q_c$ (t) | Avg Grade | $Q_r$ (g) | Profit ($) | NPV ($) |
|---|---|---|---|---|---|---|---|
| 1 | 0.500 | 100.000 | 49.500 | 0.745 | 36.900 | 160.650 | 840.800 |
| 2 | 0.500 | 100.000 | 49.500 | 0.745 | 36.900 | 160.650 | 806.265 |
| 3 | 0.500 | 100.000 | 49.500 | 0.745 | 36.900 | 160.650 | 766.550 |
| 4 | 0.500 | 100.000 | 49.500 | 0.745 | 36.900 | 160.650 | 720.890 |
| 5 | 0.500 | 100.000 | 49.500 | 0.745 | 36.900 | 160.650 | 668.371 |
| 6 | 0.500 | 100.000 | 49.500 | 0.745 | 36.900 | 160.650 | 607.977 |
| 7 | 0.500 | 100.000 | 49.500 | 0.745 | 36.900 | 160.650 | 538.524 |
| 8 | 0.497 | 100.000 | 50.000 | 0.743 | 37.150 | 164.425 | 469.430 |
| 9 | 0.483 | 97.400 | 50.000 | 0.736 | 36.800 | 161.500 | 376.612 |
| 10 | 0.467 | 94.500 | 50.000 | 0.729 | 36.425 | 158.538 | 273.221 |
| 11 | 0.450 | 91.667 | 50.000 | 0.720 | 36.000 | 154.495 | 156.594 |
| 12 | 0.450 | 16.500 | 9.000 | 0.720 | 6.480 | -189.730 | -218.190 |

## 3. Results and Discussion

Figure 4 shows a comparison of cut-off grades when dilution and waste rehabilitation costs are included in Lane's algorithm. The cut-off grades remained the same in the early parts of the mine life but decreased at a slower rate than Lane's approach. This is due to the opposing effects of dilution and waste rehabilitation costs.

Dilution increases the tonnage of materials to be mined and also decreases the average grade of a given parcel of material. To operate at the most profitable level, the cut-off grades have to be raised in order to cater for the extra cost of exploitation, loading, transportation, processing, waste dumping, and environmental issues. The algorithm therefore increases the cut-off grades as practically as possible so as to maximise the net present value. This reduces the quantity of materials sent the processing plant and increases materials dumped as waste.

Waste rehabilitation cost is also accompanied with each tonne of waste dumped. The effect of this cost is that it reduces the cut-off grade so as to reduce the quantity of materials that should be dumped as waste. Thus, penalizing waste dumping during cut-off grade optimisation makes it more economic to process materials at a reduced cut-off grade in order to reduce the amount of material dumped as waste. This forces the algorithm to decrease the cut-off grades as much as possible so as to reduce the amount of materials sent to the waste dump, as a result, more materials will be processed. The total amount of rehabilitation cost during and after ore extraction is decreased and the total achievable NPV of the project will increase.

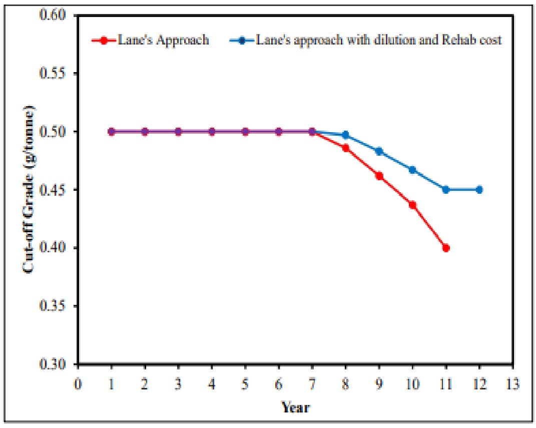


**Figure 4 Yearly Cut-Off Grades**

Due to the opposing effects of the two parameters, the modified Lane's algorithm finds a trade-off that maximises the total net present value of the project. In effect, the cut-off grades remained the same in the first half of the mine life and increased for the modified Lane's approach in the final half. At the end of the mine life, the optimum cut-off grade was pegged at 0.45 g/t against 0.40 g/t in Lane's algorithm. The life of mine also increased by one year. This is due to the extra 100 tonnes of waste that was considered as a result of dilution. Figure 5 also shows the yearly quantities sent to the processing plant.

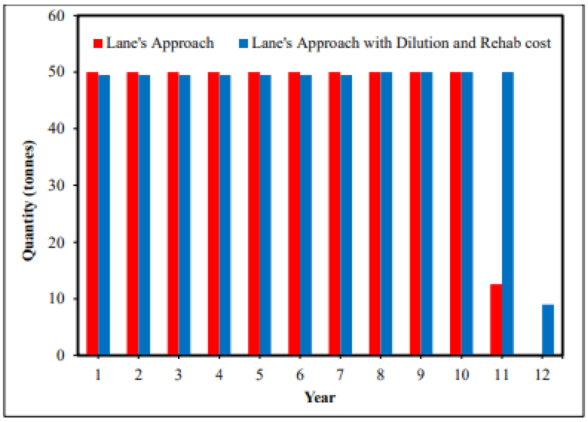


**Figure 5 Yearly Quantities Sent to the Plant**

Figure 6 shows a comparison of average grades when dilution and rehabilitation costs are included in Lane's algorithm. Due to dilution, the average grades of materials decreased in a significant portion of the mine life. At the later parts of the mine life, the average grades increased slightly above the ones recorded in Lane's algorithm. This decrease in average grades, accompanied by the constant cut-off grade in early part of the mine life, lead to a decrease in the quantities refined in those years as shown in Figure 7.

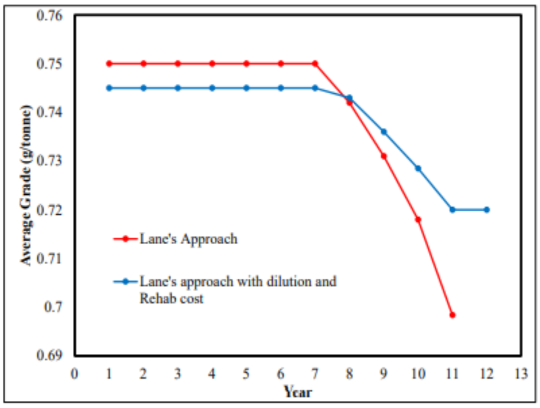


**Figure 6 Yearly Average Grades**

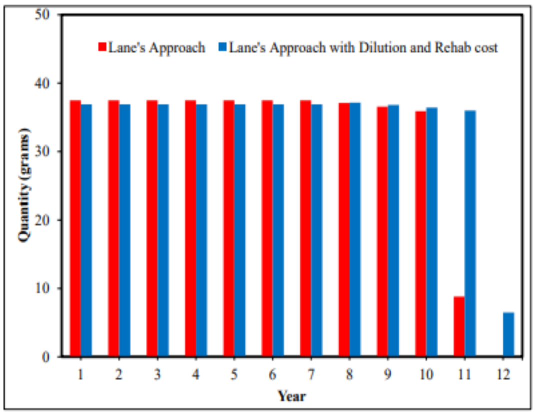


**Figure 7 Yearly Refined Quantities**

Figure 8 shows a comparison of NPVs when dilution and rehabilitation costs are included in Lane's algorithm. The NPV reduced from $ 8 395 to $ 6 007 due to dilution and rehabilitation costs as shown in Figure 8, thus dilution and rehabilitation costs reduced the NPV by $ 2 388. This is as result of the extra costs associated with dilution and waste dumping. These two costs add up to the final production cost and therefore reduces profits for a given mineral inventory.

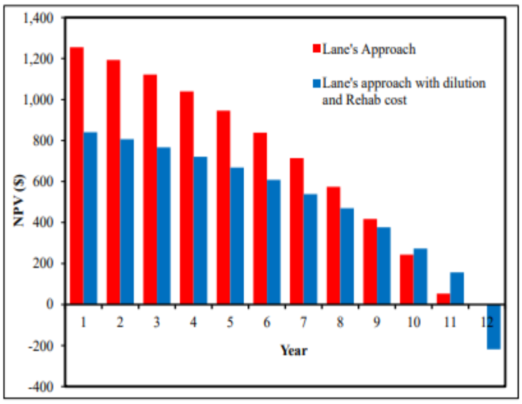


**Figure 8 Yearly NPVs**

In this paper the cost of dilution and waste rehabilitation are inserted into the optimisation process and a new mathematical model is developed based on Lane’s method**.** The authors in [3] analysed computer aided cut-off grade based on Lane’s algorithm using written codes that carried out the Lane approach as fast as possible. The author in [11] determined the optimum cut-off grade in two deposits based on Lane’s original algorithm with emphasis on the link between supplies of ores from the mine to stockpiles and from stockpiles to the processing plants. However, results in [13] introduced generalised reduced gradient to Lane's algorithm to maximise the NPV of the hypothetical project. In summary they had diverse objectives based on different data sets for the Lane's algorithm. The authors in [3;11 and 13] did not explicitly factor dilution, or rehabilitation as done in this optimisation of cut-off grade.

## 4. Conclusion

Results of the application of this model show that considering dilution and waste rehabilitation costs causes average grade to decrease in a significant portion of the life of mine; consequently, the cutoff grade increases. Quantities of materials to be mined also increase and, as a result, may increase the life of mine. Dilution and waste rehabilitation have significant effects on the annual profits and NPVs of mining projects. Dilution and rehabilitation costs reduced the NPV and annual profit by $ 2 388 (a 28% decrease) and $ 944 (a 37% decrease) respectively. Hence, they should be accounted for when determining optimal cut-off grades for open pit mines. It is therefore important to account for costs associated with dilution and waste rehabilitation in cut-off grade policies. It is also recommended that this modified Lane's approach be made to account for the volatility of the various costs associated with the policy, also include other options such as blending, stockpiling and multiple processing streams.

**Nomenclature for**
**Defining Parameters of the Model**

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| Variable | Definition | Unit of Measurement |
|---|---|---|
| *M* | The maximum amount of material (ore and waste) that a mine can produce in a given period | for example, 50.000 tons per year |
| C | The maximum amount of ore that can be processed through the concentrator in a given period. | (for example, 1000 tons pe year), assuming an unrestricted supply of input ore from the mine, producing a concentrate of a fixed-grade |
| R | The maximum amount of final product (commodity mined) produced in a set period | (for example, 400 grams/year). |
| *m* | The mining cost of material moved irrespective of material class (ore or waste). | expressed in dollars per ton |
| c | The concentrating costs. Usually comprises cost involved in crushing, grinding, floating, leaching, and other processes | expressed in dollars per ton of milled material |
| r | This includes all costs incurred during the manufacturing and selling stages, such as smelting, refining, packaging, freight, insurance, and so on. | This is given in terms of dollars per unit of product |
| *f* | The fixed cost includes all costs such as rent, administration, road and building maintenance, and so on that are independent of production levels. | It is expressed as a fixed cost over the production period under consideration (for example, one year). |
| s | This is expressed as a price per unit of product | Dollars per unit |
| *y* | The percentage of minerals in the original ore feed retained in the final product | % |
| *d* | This is the current discount rate. | % |
| Qm | The quantity of material to be mined | tons |
| $Q_c$ | The quantity of ore sent to the concentrator | tons |
| $Q_r$ | The amount of product produced over the production period. | tons |
| *T* | The length of the production period being considered | (for example, one year). |
| | | |

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