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Effect of Imperfections on the Buckling Behaviour of a Pressurized Circular-Elliptical Toroid

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Abstract

It has been reported that the presence of imperfections in a non-conventional toroidal vessel can affect the buckling response of the structure, depending mainly on the cross-sectional parameters of the shell. This paper attempts to provide information regarding the buckling sensitivity of externally pressurized circular-elliptical toroidal shells containing initial geometric imperfections in the form of eigenmodes. The toroidal vessel itself is made up of a top semi-circular toroidal segment that is joined tangentially to a bottom semi-elliptical segment at their equatorial circles of latitude. A numerical approach using the commercially available ABAQUS tool was employed for the investigation in which suitable mesh densities and boundary conditions leading to the lowest failure values were adopted throughout. Various degrees of imperfections were separately superimposed in both the tall (b/a=3.0) and short (b/a=0.5) toroids, keeping a/t=200 constant for all cases considered. While the buckling of short toroidal vessels was insensitive to initial geometric imperfections, the tall ones were found to be generally sensitive to the imperfections, with up to 70% reduction in failure loads recorded in some cases.

1. Introduction

For available space considerations, toroidal pressure tanks can provide a better option for an on-board gaseous fuel storage in vehicles and aerospace applications, compared with the traditional cylindrical pressure vessel [1, 2, 3]. It also finds wide applications in fossil and nuclear power, petrochemical, and aviation industries. Toroidal shells can be constructed to take any desired cross-sectional form, though the traditional cross-section is of a circular profile. Other cross-sectional forms are available and some of these have received considerable intention in the research community. An example of these can be found in the work of [4], where an elliptical toroidal vessel was investigated. Buckling collapse is a prominent failure mode of pressure vessels [5]. [6-8] studied buckling and deformation of loaded non-circular toroidal cross-sections such as elliptical, ovaloid and doubly symmetric parabolic–ogival cross-sections. [9] provided membrane and buckling solutions for multi-shell toroidal pressure vessel, while [10] considered a semi-elliptical toroid and presented an approximate bending solution for the shell. [8] found that the buckling strength of the toroidal vessel is strongly dependent on the geometrical parameters of the cross-section. The work also showed that the buckling behaviour is sensitive to initial geometric imperfections in the form of eigenmodes for the cross-section considered for the toroids in the paper. A numerical study into the buckling resistance of geometrically perfect and imperfect steel toroidal shells with closed cross-sections was conducted in [11], where initial geometric imperfections in the form of the eigenmodes were modelled by a cosine function. The effect of initial geometric imperfections in an externally pressurized circular-elliptical toroidal shells on the buckling behaviour is also studied in this paper using ABAQUS.

2. Methodology

The sketch of cross-sectional view of the toroidal vessel adopted in this study is shown in Figure 1, in which the loading and geometrical parameters of the shell are presented. The vessel consists of a top semi-circular toroidal segment that is joined tangentially to a bottom semi-elliptical segment at their equatorial circles of latitude (at D^o in the outer region and D^i in the inner region of the vessel). Hence, the total height of the vessel is $a+b$, where a is the local radius and the local (horizontal) semi-axis of the semi-circular and semi-elliptic toroidal segments, respectively, and b is the local (vertical) semi-axis of the semi-elliptic segment. The toroidal mean radius of the vessel is denoted by A and the entire external surface of the vessel is subjected to a uniform pressure, which is denoted by a patch pressure load p in Figure 1.

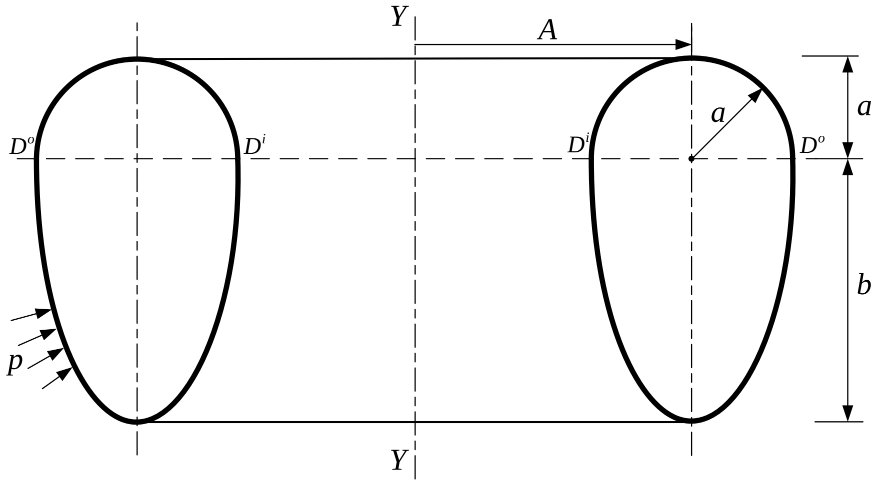

Figure 2: Externally pressurised circular-elliptic toroidal vessel

The numerical modelling procedure and assumptions employed and validated in [8] have been adopted in this present study. Here, two distinct configurations for the vessel - the short and tall toroids, based on b/a ratio - were used in the investigation. Consequently, only $a/t=200$ toroids with $b/a=0.5$ and 3.0 were considered for the short and tall toroids, respectively. The toroidal shell material was modelled as elastic steel with Young modulus $E=210\times10^9 N/m^2$ and Poisson's ratio $\upsilon=0.3$. In line with [8], appropriate mesh densities were adopted for the models of the four-node doubly curved thin shell elements (S4R), which is available in the ABAQUS FE tool. The support conditions leading to the lowest buckling values were employed in the models, see Table 1.

Table1: Boundary conditions applied at the inner-most circle of latitude

u_r	u_z	u_x	u_y	u_z	Φ_x	Φ_y	Φ_z
0	0	0	0	0	$\neq 0$	$\neq 0$	$\neq 0$

Firstly, a linear eigenvalue buckling analysis was conducted to obtain the required number of eigenmodes. The modes were characterised by either local or global circumferential and longitudinal wave numbers. By varying the imperfection amplitude (scaling factor) of the each of the modes, the selected modes were superimposed individually on the perfect toroidal shell model. This was then analysed by the modified Riks algorithm in ABAQUS to obtain the failure load of the toroids. The eigen buckling results obtained for various toroids with $b/a = 0.5$ and 3.0 are presented in the following.

2.1. Eigenshapes for toroids with b/a=3.0

A toroid with $b/a = 3.0$ is an example of the tall circular-elliptic assemblies which was found to fail by asymmetric bifurcation with $n > 0$, where n is the circumferential wave number. Each of the first ten modes obtained from the linear buckling analysis of an externally pressurised toroid with $b/a = 3.0$, $A/a = 2.0$ and $a/t = 200$, was seen to have a single longitudinal wave. The eigenmodes were very closely spaced, which suggests that they may interact and lead to a lower buckling strength of the imperfect toroid. Also, odd and even mode numbers were found to have the same eigenmodes that are only different in terms of phase due to the axis-symmetry of the present toroidal geometry, and some of the modes were found to have the same n number. Hence, the first, third and seventh eigenmodes with distinctive shapes were selected to perform checks on the sensitivity of the buckling load to eigenshape deviations from the perfect geometry. The bottom view of each of the three modes is shown in Figure 2, and a cross-sectional view can be seen in Figure 3, for example. The first, third and seventh eigenshapes are characterised by 27, 28, 29 circumferential waves, respectively. The deformations of the three modes are seen to be affine to the upper zone of the semi-elliptical segment near the junction of the circular-elliptic shells of the toroidal vessel.

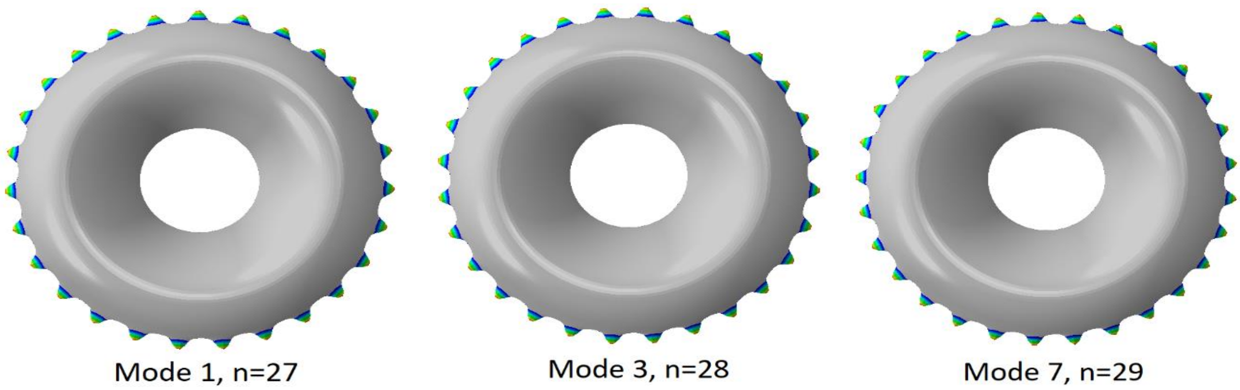

Figure 2: Bottom views of eigenmodes of a *tall* circular-elliptic toroid with $b/a = 3.0$, $A/a = 2.0$, $a/t = 200$

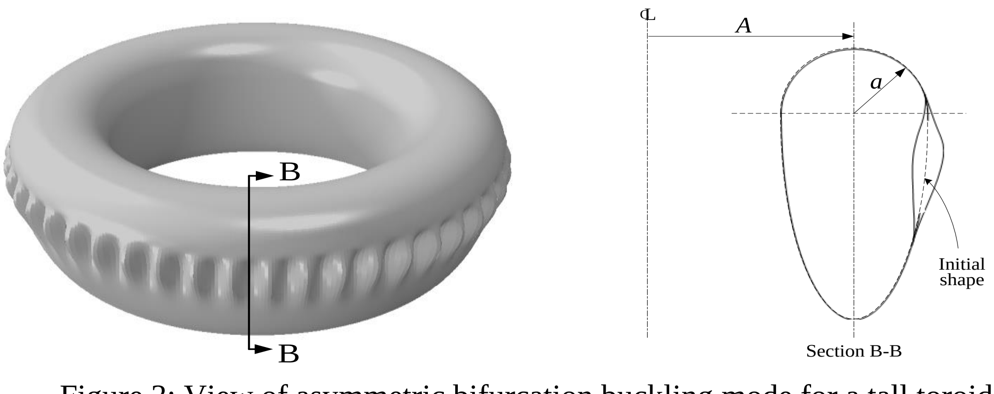

Figure 3: View of asymmetric bifurcation buckling mode for a tall toroid with $b/a = 3.0$, $A/a = 4$, $a/t = 200$, $n = 45$, $p_{cr} = 0.093MPa$

2.2. Eigenshapes for toroids with b/a=0.5

Here, the imperfection sensitivity of a $a/t = 200$ toroid with $b/a = 0.5$, which is an example of *short* circular-elliptic toroidal assemblies and usually fails with $n = 0$ is considered. An eigenvalue analysis was first conducted on the vessel with $A/a = 2$. The ensuing first, third and sixth eigenmodes with distinctive shapes were selected to perform checks on the sensitivity of the buckling load to eigenshape deviations from the perfect geometry. The bottom view of each of the three modes is shown in Figure 4. The first eigenmode of the vessel is characterised by $n = 0$, see Figure 5 (this is similar to that of the circular toroid [8]). The third mode is characterised by 2 longitudinal waves and 4 circumferential waves, while the sixth mode is also characterised by 2 longitudinal waves but 5 circumferential waves. The deformation in modes 2 and 5 are seen to be affine to the semi-elliptical segment of the toroidal vessel.

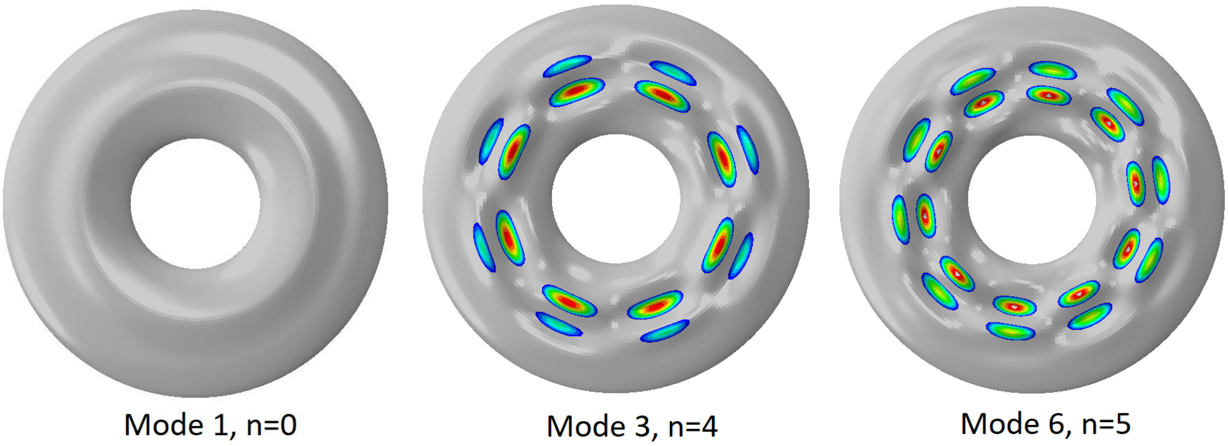

Figure 4: Bottom view of eigenmode of a *short* circular-elliptic toroid with $b/a = 0.5$, $A/a = 2.0$, $a/t = 200$

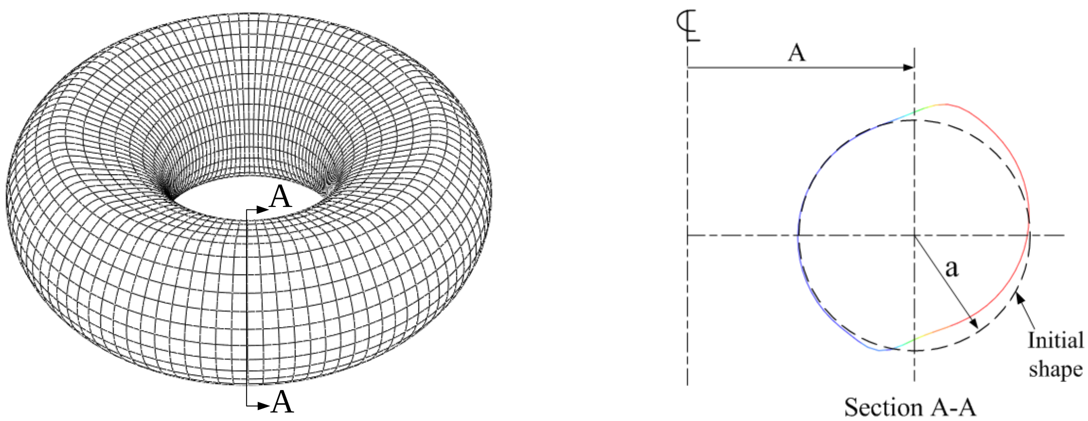

Figure 5: View of axisymmetric bifurcation buckling mode for a short toroid with $b/a = 1$, $A/a = 2$, $a/t = 200$, $n = 0$, $p_{cr} = 0.108 MPa$

3.0. Results and Discussion

For the selected eigenmodes, different scaling factors of the shape deviation of each of the three eigenmodes were separately superimposed on perfect short and tall circular-elliptic toroidal vessels, to assess the imperfection sensitivity of the toroids within the ABAQUS framework. The results for the tall and short toroids are shown for a range of modulated imperfection amplitude to the average wall thickness ratio Δ/t, between 0.0 to 1.0. This is presented in the following:

3.1. Effect of imperfection sensitivity of tall toroids with b/a=3.0

The effect of imperfection amplitude Δ on the collapse pressure of the toroidal vessel, using the Riks method, is shown in Figure 6 based on each of the buckling modes 1, 3 and 7 of Figure 2. Figure 6 shows the ratio of critical buckling pressures of imperfect toroids to that of corresponding perfect toroids (p_{cr}^{imp} / p_{cr}) against modulated imperfection amplitude to average wall thickness ratio (Δ/t). The results indicate that the toroid with $b/a = 3.0$, $A/a = 2.0$ and $a/t = 200$ is sensitive to initial eigenmode type geometric imperfections within the modulated amplitude range of $0 \le \Delta/t \le 1.0$.

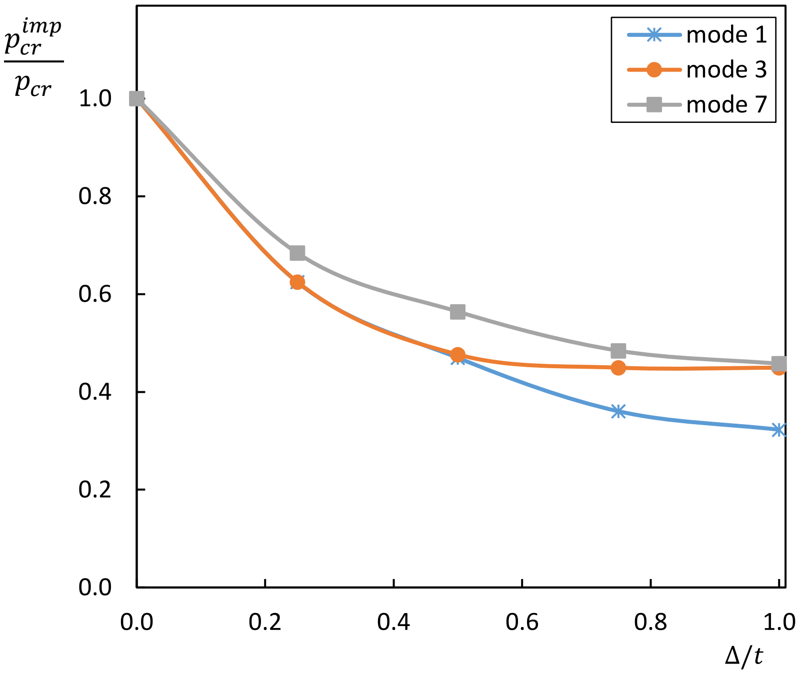

Figure 6: Effect of imperfection amplitude of various eigenmodes on the buckling pressures of toroids with $b/a = 3.0$, $A/a = 2.0$, $a/t = 200$

The investigation was also extended for various values of A/a. The imperfection sensitivity of toroids with three different values of A/a and a single value of each of $b/a = 3.0$ and $a/t = 200$ were conducted by superimposing various amplitude of imperfection of only the first eigenmode. On account of 'lower bound' concept of the lowest practically achievable buckling load, the first mode shape was easily chosen from the cases studied above since it leads to the largest imperfection reduction of buckling strength of the toroidal shells. Typical buckling results for each of the toroids with A/a = 1.5, 2.0 and 4.0 were plotted against various degrees of the imperfection of mode 1. This is shown in Figure 7, indicating that, even for different A/a values, initial imperfections in the toroids affect the buckling response of the tall shells.

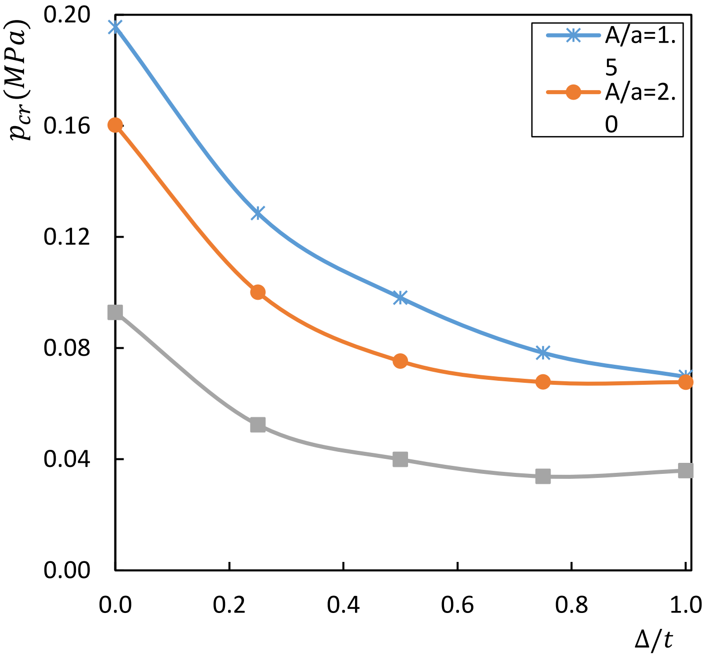

Figure 7: Sensitivity of critical buckling load to various amplitudes of first eigenmode imperfection in toroids with $b/a = 3.0$, $a/t = 200$ and different A/a ratios

From the results and plots in Figures 6 and 7, it can be seen that the buckling pressures of *tall* circular-elliptic toroidal vessels with $a/t = 200$, which usually fail asymmetrically with $n > 0$, are greatly reduced by the presence of initial geometric-type imperfections with scaling factor Δ of up to the thickness of the toroidal vessels. The behaviour of the tall circular-elliptic toroidal shells is similar to that of complete toroids of prolate elliptical cross-section [8, 12].

3.3. Effect of imperfection sensitivity of short toroids with b/a=0.5

The effect of imperfection amplitude Δ on the collapse pressure of the toroidal vessel, using Riks method in ABAQUS FE, is shown in Figure 8 based on each of the buckling modes 1, 3 and 6 of Figure 4. Figure 8 shows the ratio of critical buckling pressures of imperfect toroids to that of perfect toroids (p_{cr}^{imp} / p_{cr}) against modulated imperfection amplitude to average wall thickness ratio (Δ/t). The results indicate that the short toroids with $b/a = 0.5$, $A/a = 2.0$ and $a/t = 200$ are not be sensitive to initial eigenmode-type geometric imperfections within the modulated amplitude in the range $0 \le \Delta/t \le 1.0$.

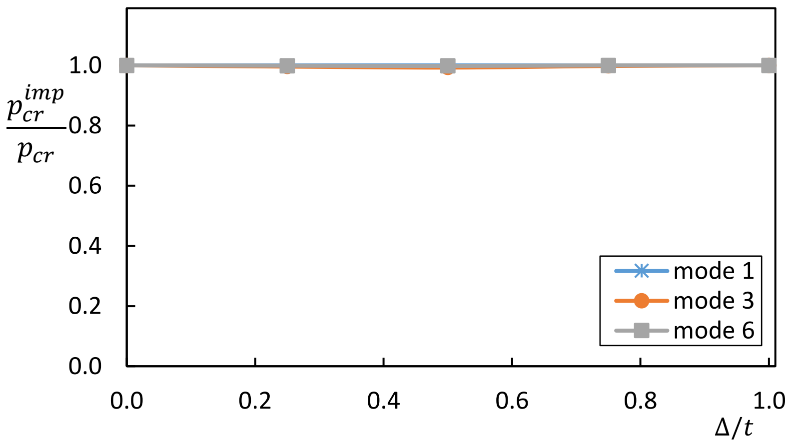

Figure 8: Effect of imperfection amplitude of various eigenmodes on the buckling pressures of toroids with $b/a = 0.5$, $A/a = 2.0$, $a/t = 200$

Toroids with various A/a were also considered. The imperfection sensitivity of toroids with three different values A/a and a single value of each of $b/a = 0.5$ and $a/t = 200$ were also investigated by superimposing various amplitudes of imperfection of only the first eigenmode. Typical buckling results for each of the toroids with A/a = 1.5, 2.0 and 4.0 are plotted against various degrees of the imperfection of mode 1, are shown in Figure 9. This, again, shows that toroidal shells of this type are not sensitive to a single eigenmode imperfection with a modulated amplitude of $0 \le \Delta/t \le 1.0$.

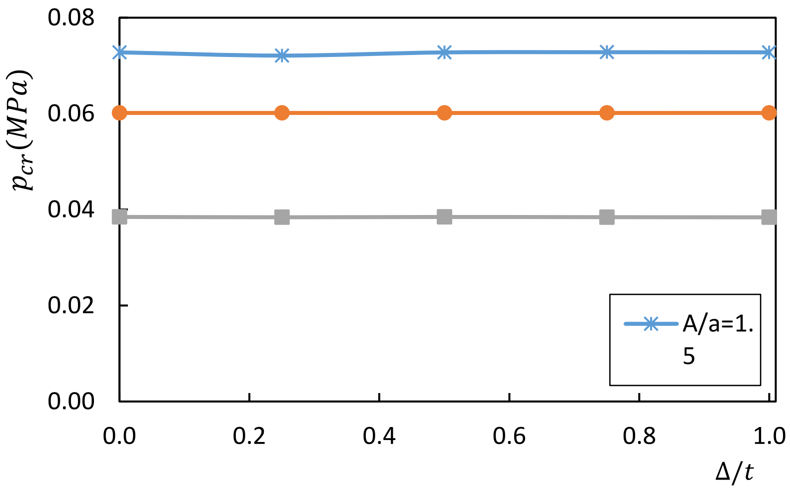

Figure 9: Sensitivity of critical buckling load to various amplitudes of first eigenmode imperfection in toroids with $b/a = 0.5$, $a/t = 200$ and different A/a ratios

From the results and plots in Figure 8 and 9, one can infer that that the buckling pressures of *short* circular-elliptic toroidal vessels, which usually fail axisymmetrically with $n = 0$, are negligibly affected by the presence of initial geometric imperfections with scaling factor Δ of up to the thickness of the toroidal vessels. This is not the case for most structural shell elements. [12] have shown that the critical loads of circular toroids (which of course, fall within the range of the short toroids under discussion), are not too sensitive to eigenshape imperfections. A quick look on the spacing of the eigenmodes of some of these short toroids, further buttresses that they are likely not affected by initial imperfections of eigenmode types, as there is a very wide difference between the first buckling mode and the other modes (which appear to be close to each other though). Other imperfection types and larger scaling factors of eigenshape imperfections, including the superimposition of more than one eigenmode may have interesting effects on the buckling pressures of these vessels.

4. Conclusion

This paper studied the effect of initial geometric imperfections in an externally pressurized circular-elliptical toroidal shells on the buckling behaviour using the commercially available numerical tool, ABAQUS FE. Two distinct configurations for the vessel - the short and tall toroids, based on b/a ratio - were adopted in the investigation. For each of the these, various scaling factors of the shape deviation of each of the selected eigenmodes were separately superimposed on the respective circular-elliptic toroidal vessels to assess the imperfection sensitivity of the toroids. For the tall circular-elliptic toroidal assemblies, the eigenmodes of the pressurised vessels were found to be closely spaced. The vessels were generally imperfection sensitive even for the different opening ratios A/a investigated, with up to 70% reduction in failure load recorded in some cases within a modulated imperfection amplitude range of $0 \le \Delta/t \le 1.0$. On the other hand, for the short circular-elliptic toroidal assemblies, the study on the sensitivity of the toroid under uniform external pressure generally shows that the buckling strength of a perfect short circular-elliptic toroid is not substantially affected by the presence of eigenmode-type initial imperfections.

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